Lecture 17
Semiconductors

Kittel chapter 8

Semi-Classical Equation of Motion

- $\hbar \, d\mathbf{k}/dt = F$

  Much like Newton's law, except for "crystal momentum."

- Energy conservation from the EOM

  $d\varepsilon(\mathbf{k}) / dt = \mathbf{v} \cdot F$

  $\mathbf{v} = \text{grad}_{\mathbf{k}} \, \varepsilon(\mathbf{k}) / \hbar \quad \rightarrow$ group velocity

- Derived and NOT always equivalent:

  $m^* d\mathbf{v}/dt = F$

  $m^* = \hbar^2 [d^2\varepsilon(k) / dk^2]^{-1}$

  This form is NOT as fundamental as the above, but OK for semiconductors. E.g., it cannot be used if $\varepsilon(\mathbf{k})$ is linear (as in metal).

Equation of Motion for Transport

- Simple relaxation time approximation as we discussed and used for metal

$$\hbar\,(d/dt + 1/\tau)\,\delta\mathbf{k} = \mathbf{F}$$

or

$$m^*\,(d/dt + 1/\tau)\,\mathbf{v}_d = \mathbf{F}$$

- $\delta\mathbf{k}$ is a net displacement of the whole
- The first form is more fundamental
- The second form simply defines $\mathbf{v}_d$ by $\hbar\,\delta\mathbf{k} = m^*\,\mathbf{v}_d$ where m^* is an "effective mass"

General (also for metal)

General Results from SC-EOM

- Bravais Lattice of ions gives perfect conductivity – the source of a finite conductivity is imperfections, phonons, other electrons.
- Completely filled bands or completely empty bands are as good as nothing. $\Leftrightarrow$ Only partially filled bands do things like conduct electricity and heat. (This is why a FS is so important! Distinction between metal and non-metals.)
- A few electrons missing from a completely filled band are best described by "holes."

Hole

- As in metal, consider the vacuum as the T=0 state – hole is a fermionic excitation of the valence band, completely full at T=0
- EOM $\hbar\, d\mathbf{k}/dt = \mathbf{F}$ was derived for an electron. When a band is full and there is one electron missing, however, it is better to think of a "hole."
  1. The energy, (crystal or angular) momentum, and charge for a hole is the opposite of that for an electron.
  2. The righ hand side (force) has to change sign from e to h.
  3. The group velocity of a hole is the same as the group velocity of an electron.
- The motion of all electrons except one is more correctly discussed as the motion of one hole and nothing else.

Effective mass

1. At band maximum or minimum, the energy band can be written as $\propto k^2/m^*$, where m^* is the "effective mass," which can be defined as $\hbar^2[d^2\varepsilon(\mathbf{k})/dk^2]^{-1}$.
2. The definition above is useful mainly for semiconductors and semi-metals.
3. m^* tends to be small ~ a tenth of the mass of free electron (i.e. actual excitation is lighter than free electron). [HW]
4. For metals, energy dispersion near E_F is approximated as linear – effective mass m^* is then defined in terms of v_F ($v_F = v_{F0}/m^*$) and it tends to be larger than the mass of free electron (i.e. actual excitation is heavier than free electron), due to the electron-electron interaction or a "tight-binding-like" nature of band, both important for TM or RE materials.

General (also for metal)

Charge Carriers in a Semiconductor

- Electrons in conduction band
- Holes in valence band
- Intrinsic Semiconductors: Carriers are created by excitation across the energy gap E_G
- Extrinsic Semiconductors: Carriers are provided by impurities (donors, acceptors)

Donors and Acceptors

- Consider Si or Ge, both with 4 valence electrons
- Substitutional impurities with 5 valence electrons (P, As) are donors
- Substitutional impurities with 3 valence electrons (B, Al) are acceptors
- Donors (acceptors) create impurity states just below (above) the conduction (valence) band, at energy ~ O(10) meV

Charge Carrier Population (General)

- Define energy 0 = top of valence band
- Electron density: $\mathbf{n = N_c \exp(\beta(\mu - E_G))}$

  $N_c = 2(m_e k_B T/2\pi\hbar^2)^{3/2}$
- Hole density: $\mathbf{p = N_v \exp(-\beta\mu)}$

  $N_v = 2(m_h k_B T/2\pi\hbar^2)^{3/2}$
- $np = N_c N_v \exp(-\beta E_G) = n_i p_i = n_i(T)^2 = p_i(T)^2$

  Law of mass action

Charge Carrier Population (Intrinsic Semiconductors)

- $n_i = p_i$
- $n_i = p_i = (N_c N_v)^{1/2} \exp(-\beta E_G/2)$
- Intrinsic carrier density:

  $n_i = p_i \sim 10^{16}$ (Si) and 10^{19} (Ge) m^{-3}

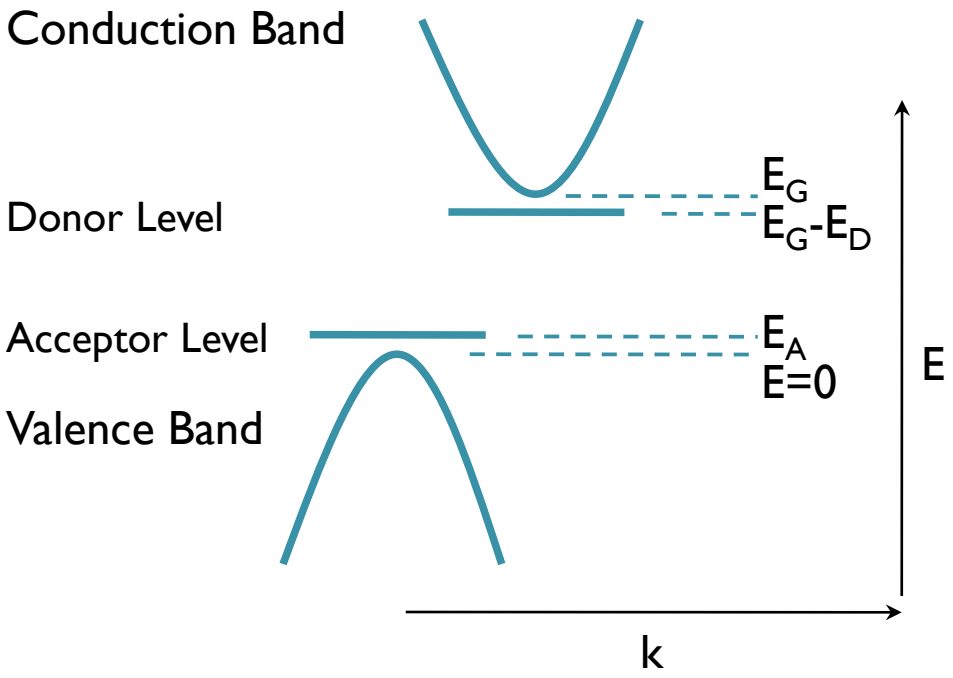
  (fewer than in metals by ~ 10 orders of magnitude or more)

- Chemical Potential for Intrinsic Semi-cond:

  $\mu = \frac{1}{2} E_G + \frac{3}{4} k_B T \ln(m_h/m_e)$

Warning: In semi-conductor literature, the chemical potential is very often referred to as the "fermi level." This is an unfortunate but this is life! Note that there is no Fermi surface there!

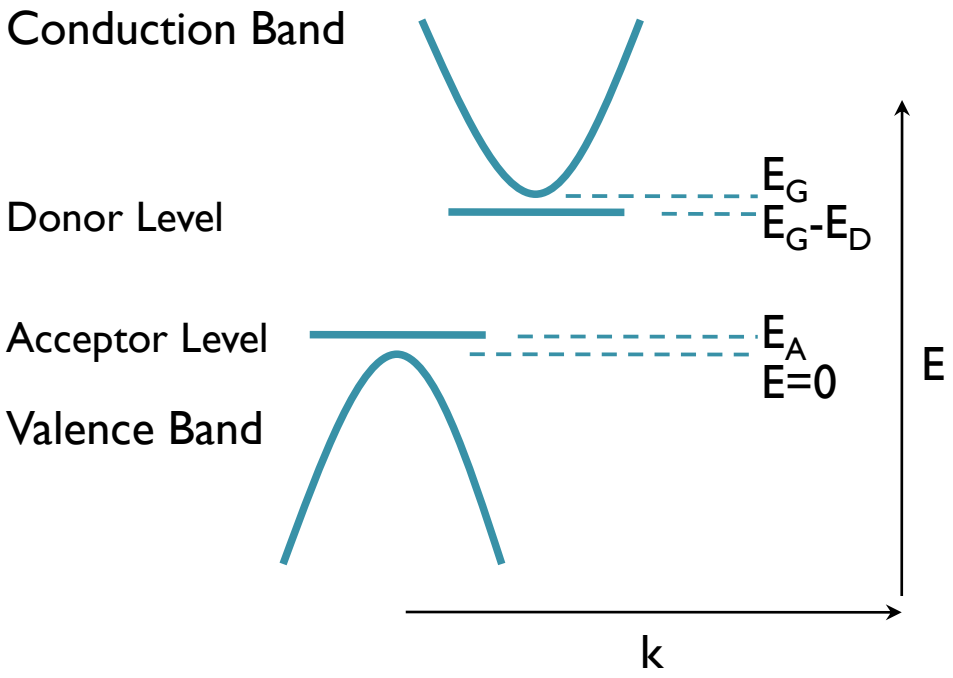
Charge Carrier Population (Extrinsic Semiconductors)



At T = 0

- Conduction Band A. is full B. is empty C. can have some electrons
- Valence Band is full
- Donors may have some electrons to spare, acceptors holes

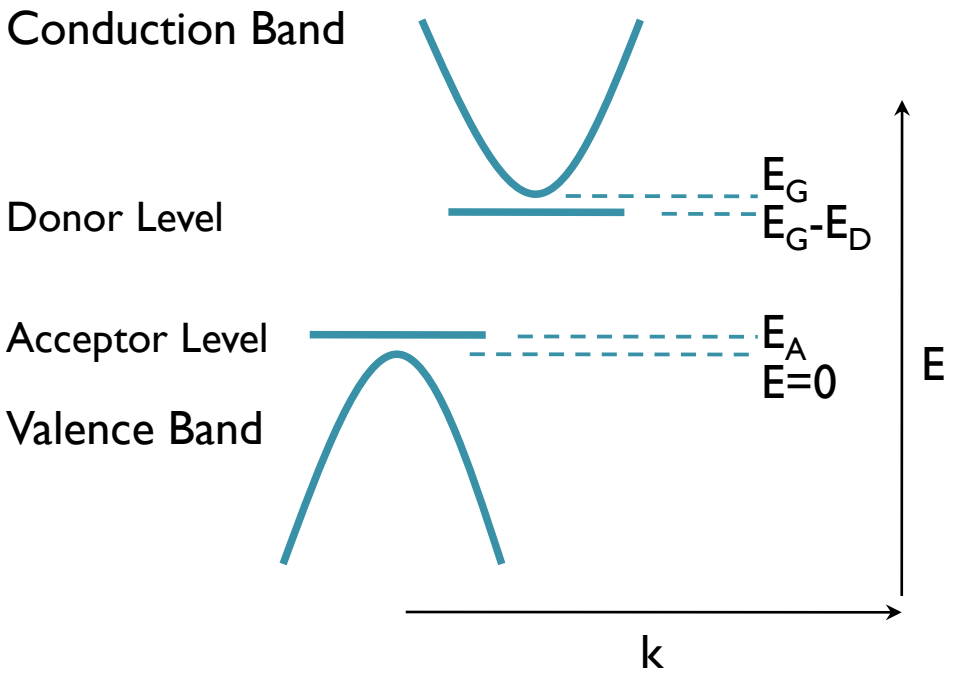
Charge Carrier Population (General)



At T = 0

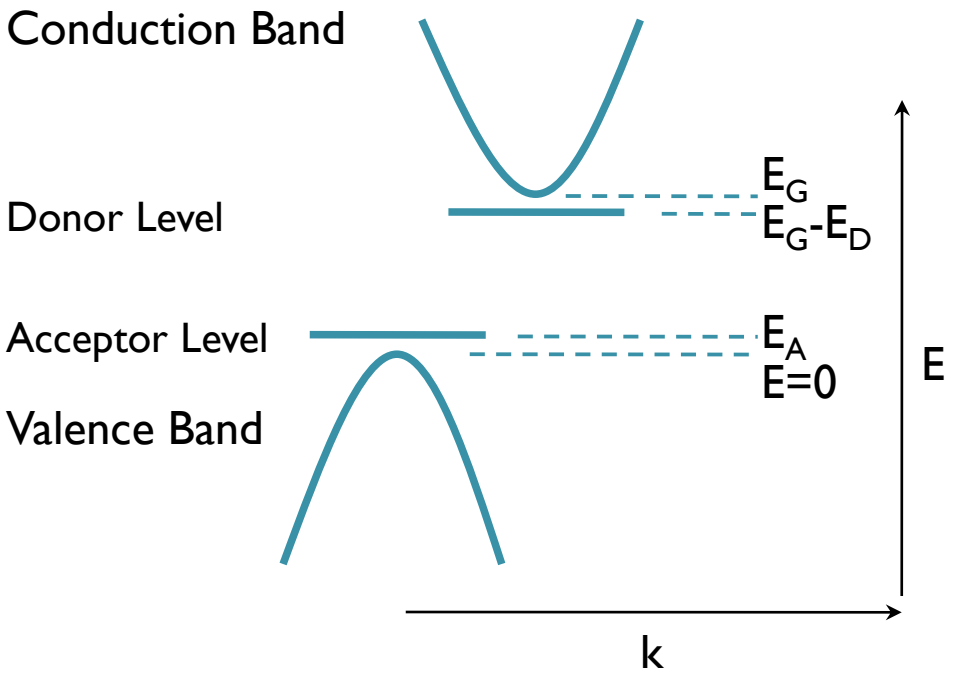
- Conduction Band is empty
- Valence Band is full
- Donors may have some electrons to spare, acceptors holes

Charge Carrier Population
Where is Chemical Potential at T=0?



- Common n-type (More donors than acceptors; majority carriers are electrons from donors, and minority carriers are holes from acceptors): A. $E_G/2$ B. E_G-E_D C. E_A
- Common p-type: E_A
- Pure n-type: A. $E_G/2$ B. E_G-E_D C. E_G-0.5E_D
- Pure p-type: $E_A/2$
- Intrinsic: $E_G/2$

Charge Carrier Population
Where is Chemical Potential at T=0?



- Common n-type (More donors than acceptors; majority carriers are electrons from donors, and minority carriers are holes from acceptors): E_G-E_D
- Common p-type: E_A
- Pure n-type: E_G-0.5E_D
- Pure p-type: E_A/2
- Intrinsic: E_G/2

Extrinsic Semiconductors (n-type)

- For $T \approx 0$

  $\mu \approx E_G - E_D$ (common n-type: small # of acceptor impurities)

  $E_G - 0.5\ E_D$ (pure n-type: absolutely no acceptors)

  $n \approx N_c \exp(-\beta E_D)$ (common n-type)

  $N_c \exp(-\beta E_D/2)$ (pure n-type)
- $n >> n_i$: electron is **majority** carrier
- $p << p_i << n$ (recall $np = n_i p_i$): hole is **minority** carrier
- As T increases ($\sim E_D$), all donors lose electrons

  $n \approx N_D - N_A$ (N_A: acceptor # density, N_D: donor # density)

  $\mu \approx E_G - k_B T \ln(N_c / (N_D - N_A)) \equiv$ ☆
- Intrinsic behaviors : at even higher T
- $\mu : E_G - [0.5]\ E_D \rightarrow$ ☆ $\rightarrow 0.5\ E_G$ as $T \uparrow$
- $n(T) : \exp(-\beta\ [0.5]\ E_D) \rightarrow N_D - N_A \rightarrow \exp(-\beta\ 0.5\ E_G)$

Extrinsic Semiconductors (p-type)

- For $T \approx 0$

  $\mu \approx E_A$ (common p-type: small # of donor impurities)

  $0.5\ E_A$ (pure p-type: absolutely no donors)

  $p \approx N_v \exp(-\beta E_A)$ (common p-type)

  $N_v \exp(-\beta E_A/2)$ (pure p-type)
- $p >> p_i$: hole is **majority** carrier
- $n << n_i << p$ ($np = n_i p_i$): electron is **minority** carrier
- As T increases ($\sim E_A$), all acceptors lose holes

  $p \approx N_A - N_D$

  $\mu \approx k_B T \ln(N_v / (N_A - N_D)) \equiv$ ☆
- Intrinsic behaviors : at even higher T
- μ : [0.5] $E_A \rightarrow$ ☆ $\rightarrow 0.5\ E_G$ as T ↑
- n(T) : $\exp(-\beta\ [0.5]\ E_A) \rightarrow N_A - N_D \rightarrow \exp(-\beta\ 0.5\ E_G)$

Example of temperature dependence of an n-type semi-cond

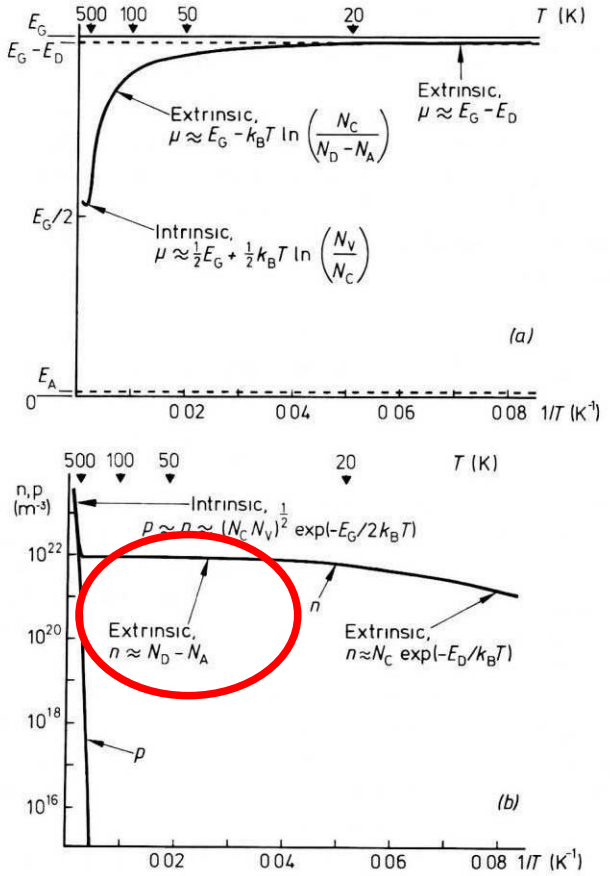


Fig. 5.6 Variations of (*a*) the Fermi level μ and (*b*) the electron and hole concentrations (note the logarithmic scale) with $1/T$ for an n-type semiconductor containing a significant number of acceptor impurities. The figure was calculated for a germanium semiconductor with $N_D = 10^{22}\ m^{-3}$, $E_D = 0.012$ eV, $N_A = 10^{21}\ m^{-3}$ and $E_A = 0.010$ eV; the scale at the top shows temperature values for this case

Transport Properties ("hole" is real)

- Equation of Motion: $m^* (d/dt + 1/\tau) \mathbf{v}_d = \mathbf{F}$ (as we used in metal)
- Hall Effect (for single type of carrier) in B field

  $R_H \equiv E_y/(B_z j_x) = -1/(ne)$ or $1/(pe)$ [sign !]
- Thermo-electric Effect can probe the sign of the carrier charge as well
- Cyclotron Resonance in B field: $\omega_c = eB/m^*$ (SI unit)
- Conductivity

  $\sigma = ne\mu_e + pe\mu_h$

  $\mu_e = e\tau_e/m^*_e = v_{d,e}/E$, and similarly for μ_e

  μ_e, μ_h: mobility
  1. useful concept for semiconductors
  2. characterizes "quality" rather than "quantity" (n or p)
  3. not important for T dependence of σ (determined by exponentials in n or p – see slide "Charge Carrier Population")